NSS

Interval Valued Neutrosophic Soft Topological Spaces

Anjan Mukherjee[1], Mithun Datta[2], Florentin Smarandache[3]

[1] Department of Mathematics, Tripura University, Suryamaninagar, Agartala-799022, Tripura, India, Email: anjan2002_m@yahoo.co.in

[2] Department of Mathematics, Tripura University, Suryamaninagar, Agartala-799022, Tripura, India, Email: mithunagt007@gmail.com

[3]Department of Mathematics, University of New Mexico, Gallup, USA, Email: smarand@unm.edu

Abstract. In this paper we introduce the concept of interval valued neutrosophic soft topological space together with interval valued neutrosophic soft finer and interval valued neutrosophic soft coarser topology. We also define interval valued neutrosophic interior and closer of an interval valued neutrosophic soft set. Some theorems and examples are cites. Interval valued neutrosophic soft subspace topology are studied. Some examples and theorems regarding this concept are presented..

Keywords: Soft set, interval valued neutrosophic set, interval valued neutrosophic soft set, interval valued neutrosophic soft topological space.

1 Introduction

In 1999, Molodtsov [9] introduced the concept of soft set theory which is completely new approach for modeling uncertainty. In this paper [9] Molodtsov established the fundamental results of this new theory and successfully applied the soft set theory into several directions. Maji et al. [7] defined and studied several basic notions of soft set theory in 2003. Pie and Miao [11], Aktas and Cagman [1] and Ali et. al. [2] improved the work of Maji et al [7]. The intuitionistic fuzzy set is introduced by Atanaasov [4] as a generalization of fuzzy set [15] where he added degree of non-membership with degree of membership. Neutrosophic set introduced by F. Smarandache in 1995 [12]. Smarandache [13] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconstant data. Maji [8] combined neutrosophic set and soft set and established some operations on these sets. Wang et al. [14] introduced interval neutrosophic sets. Deli [6] introduced the concept of interval-valued neutrosophic soft sets.

In this paper we form a topological structure on interval valued neutrosophic soft sets and establish some properties of interval valued neutrosophic soft topological space with supporting proofs and examples.

2 Preliminaries

In this section we recall some basic notions relevant to soft sets, interval-valued neutrosophic sets and interval-valued neutrosophic soft sets.

Definition 2.1: [9] Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. Then the pair (f, A) is called a *soft set* over U, where f is a mapping given by $f: A \to P(U)$.

Definition 2.2: [13] A neutrosophic set A on the universe of discourse U is defined as $A = \{(x, \mu_A(x), \gamma_A(x), \delta_A(x)) : x \in U\}$, where $\mu_A, \gamma_A, \delta_A : U \to]^-0,1^+[$ are functions such that the condition: $\forall x \in U,\ ^-0 \le \mu_A(x) + \gamma_A(x) + \delta_A(x) \le 3^+$ is satisfied.

Here $\mu_A(x), \gamma_A(x), \delta_A(x)$ represent the truth-membership, indeterminacy-membership and falsity-membership respectively of the element $x \in U$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0,1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0,1]$.

Definition 2.3: [14] An *interval valued neutrosophic set* A on the universe of discourse U is defined as $A = \{(x, \mu_A(x), \gamma_A(x), \delta_A(x)) : x \in U\}$, where $\mu_A, \gamma_A, \delta_A : U \to Int]^-0,1^+[$ are functions such that the

condition:
$\forall x \in U$, $^{-}0 \le sup\mu_A(x) + sup\gamma_A(x) + sup\delta_A(x) \le 3^{+}$ is satisfied.

In real life applications it is difficult to use interval valued neutrosophic set with interval-value from real standard or non-standard subset of $Int(]^{-}0,1^{+}[)$. Hence we consider the interval valued neutrosophic set which takes the interval-value from the subset of $Int([0,1])$ (where $Int([0,1])$ denotes the set of all closed sub intervals of $[0,1]$). The set of all interval valued neutrosophic sets on U is denoted by *IVNS(U).*

Definition 2.4: [6] Let U be an universe set, E be a set of parameters and $A \subseteq E$. Let *IVNs(U)* denotes the set of all interval valued neutrosophic sets of U. Then the pair (f, A) is called an *interval valued neutrosophic soft set* (*IVNSs* in short) over U, where f is a mapping given by $f : A \to IVNs(U)$. The collection of all interval valued neutrosophic soft sets over U is denoted by *IVNSs(U).*

Definition 2.5: [6] Let U be a universe set and E be a set of parameters. Let $(f, A), (g, B) \in IVNSs(U)$, where $f : A \to IVNs(U)$ is defined by

$$f(a) = \left\{\left(x, \mu_{f(a)}(x), \gamma_{f(a)}(x), \delta_{f(a)}(x)\right) : x \in U\right\}$$

and $g : B \to IVNs(U)$ is defined by

$$g(b) = \left\{\left(x, \mu_{g(b)}(x), \gamma_{g(b)}(x), \delta_{g(b)}(x)\right) : x \in U\right\}$$

where

$\mu_{f(a)}(x), \gamma_{f(a)}(x), \delta_{f(a)}(x), \mu_{g(b)}(x), \gamma_{g(b)}(x), \delta_{g(b)}(x) \in Int([0,1])$

for $x \in U$. Then

(i) (f, A) is called *interval valued neutrosophic subset* of (g, B) (denoted by $(f, A) \subseteq (g, B)$) if $A \subseteq B$ and $\mu_{f(e)}(x) \le \mu_{g(e)}(x)$, $\gamma_{f(e)}(x) \ge \gamma_{g(e)}(x)$, $\delta_{f(e)}(x) \ge \delta_{g(e)}(x)$ $\forall e \in A, \forall x \in U$. Where

$\mu_{f(e)}(x) \le \mu_{g(e)}(x)$ iff $inf\,\mu_{f(e)} \le inf\,\mu_{g(e)}$ and $sup\mu_{f(e)} \le sup\mu_{g(e)}$

$\gamma_{f(e)}(x) \ge \gamma_{g(e)}(x)$ iff $inf\,\gamma_{f(e)} \ge inf\,\gamma_{g(e)}$ and $sup\gamma_{f(e)} \ge sup\gamma_{g(e)}$

$\delta_{f(e)}(x) \ge \delta_{g(e)}(x)$ iff $inf\,\delta_{f(e)} \ge inf\,\delta_{g(e)}$ and $sup\delta_{f(e)} \ge sup\delta_{g(e)}$.

(ii) Their *union*, denoted by $(f, A) \cup (g, B) = (h, C)$ (say), is an interval valued neutrosophic soft set over U, where $C = A \cup B$ and for $e \in C$, $h : C \to IVNS(U)$ is defined by

$h(e) = \left\{\left(x, \mu_{h(e)}(x), \gamma_{h(e)}(x), \delta_{h(e)}(x)\right) : x \in U\right\}$, where for $x \in U$,

$$\mu_{h(e)}(x) = \begin{cases} \mu_{f(e)}(x) & if\ e \in A - B \\ \mu_{g(e)}(x) & if\ e \in B - A \\ \mu_{f(e)}(x) \vee \mu_{g(e)}(x) & if\ e \in A \cap B \end{cases}$$

$$\gamma_{h(e)}(x) = \begin{cases} \gamma_{f(e)}(x) & if\ e \in A - B \\ \gamma_{g(e)}(x) & if\ e \in B - A \\ \gamma_{f(e)}(x) \wedge \gamma_{g(e)}(x) & if\ e \in A \cap B \end{cases}$$

$$\delta_{h(e)}(x) = \begin{cases} \delta_{f(e)}(x) & if\ e \in A - B \\ \delta_{g(e)}(x) & if\ e \in B - A \\ \delta_{f(e)}(x) \wedge \delta_{g(e)}(x) & if\ e \in A \cap B \end{cases}$$

(iii) Their *intersection*, denoted by $(f, A) \cap (g, B) = (h, C)$ (say), is an interval valued neutrosophic soft set of over U, where $C = A \cap B$ and for $e \in C$, $h : C \to IVNS(U)$ is defined by

$h(e) = \left\{\left(x, \mu_{h(e)}(x), \gamma_{h(e)}(x), \delta_{h(e)}(x)\right) : x \in U\right\}$, where for $x \in U$ and $e \in C$,

$\mu_{h(e)}(x) = \mu_{f(e)}(x) \wedge \mu_{g(e)}(x), \gamma_{h(e)}(x) = \gamma_{f(e)}(x) \vee \gamma_{g(e)}(x)$ and $\delta_{h(e)}(x) = \delta_{f(e)}(x) \vee \delta_{g(e)}(x)$.

(iv) The *complement* of (f, A), denoted by $(f, A)^c$ is an interval valued neutrosophic soft set over U and is defined as $(f, A)^c = (f^c, \rceil A)$, where $f^c : \rceil A \to IVNS(U)$ is defined by

$$f^c(a) = \left\{\left(x, \delta_{f(a)}(x), \left[1 - sup\gamma_{f(a)}(x), 1 - inf\,\gamma_{f(a)}(x)\right], \mu_{f(a)}(x)\right) : x \in U\right\}$$

for $a \in A$.

Definition 2.6:[5,6] An *IVNSs* (f, A) over the universe U is said to be universe *IVNSs* with respect to A if $\mu_{f(a)}(x) = [1,1]$, $\gamma_{f(a)}(x) = [0,0]$, $\delta_{f(a)}(x) = [0,0]$ $\forall x \in U, \forall a \in A$. It is denoted by I.

Definition 2.7: An *IVNSs* (f,A) over the universe U is said to be null *IVNSs* with respect to A if $\mu_{f(a)}(x)=[0,0]$, $\gamma_{f(a)}(x)=[1,1]$, $\delta_{f(a)}(x)=[1,1]$ $\forall x\in U,\forall a\in A$. It is denoted by ϕ.

3 Interval Valued Neutrosophic Soft Topological Spaces

In this section, we give the definition of interval valued neutrosophic soft topological spaces with some examples and results. We also define discrete and indiscrete interval valued neutrosophic soft topological space along with interval valued neutrosophic soft finer and coarser topology.

Let U be an universe set, E be the set of parameters, $\wp(U)$ be the set of all subsets of U, *IVNs(U)* be the set of all interval valued neutrosophic sets in U and *IVSNs(U;E)* be the family of all interval valued neutrosophic soft sets over U via parameters in E.

Definition 3.1: Let (ζ_A,E) be an element of *IVNSs(U;E)*, $\wp(\zeta_A,E)$ be the collection of all interval valued neutrosophic soft subsets of (ζ_A,E). A sub family τ of $\wp(\zeta_A,E)$ is called an interval valued neutrosophic soft topology (in short *IVNS*-topology) on (ζ_A,E) if the following axioms are satisfied:

(i) $(\phi_{\zeta_A},E),(\zeta_A,E)\in\tau$

(ii) $\{(f_A^k,E):k\in K\}\subseteq\tau\Rightarrow\bigcup_{k\in K}(f_A^k,E)\in\tau$

(iii) If $(g_A,E),(h_A,E)\in\tau$ then $(g_A,E)\cap(h_A,E)\in\tau$

The triplet (ζ_A,E,τ) is called interval valued neutrosophic soft topological space (in short *IVNS*-topological space) over (ζ_A,E). The members of τ are called τ –open *IVNS* sets (or simply open sets). Here $\phi_{\zeta_A}:A\to IVNS(U)$ is defined as $\phi_{\zeta_A}(e)=\{(x,[0,0],[1,1],[1,1]):x\in U\}$ $\forall e\in A$.

Example 3.2: Let $U=\{u_1,u_2,u_3\}$, $E=\{e_1,e_2,e_3,e_4\}$, $A=\{e_1,e_2,e_3\}$. The tabular representation of (ζ_A,E) given by

U	e_1	e_2	e_3
u_1	([.5,.8],[.3,.5],[.2,.7])	([.4,.7],[.2,.3],[.1,.3])	([.3,.9],[0,.1],[0,.2])
u_2	([.4,.7],[.3,.4],[.1,.2])	([.6,.9],[.1,.2],[.1,.2])	([.4,.8],[.1,.2],[0,.5])
u_3	([.5,1],[0,.1],[.3,.6])	([.6,.8],[.2,.4],[.1,.3])	([.4,.9],[.1,.3],[.2,.4])

Table1:Tabular representation of (ζ_A,E)

The tabular representation of (ϕ_{ζ_A},E) is given by

U	e_1	e_2	e_3
u_1	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	*([0,0],[1,1],[1,1])*
u_2	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	*([0,0],[1,1],[1,1])*
u_3	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	*([0,0],[1,1],[1,1])*

Table2:Tabular representation of (ϕ_{ζ_A},E)

The tabular representation of (f_A^1,E) is given by

U	e_1	e_2	e_3
u_1	*([.1,.7],[.4,.8],[.3,1])*	*([.1,.3],[.4,.6],[.2,.6])*	*([.2,.5],[.8,.9],[.4,.9])*
u_2	*([.1,.3],[.6,.7],[.2,.8])*	*([0,.5],[.5,.8],[.4,1])*	*([0,.3],[.6,.9],[.1,.7])*
u_3	*([.4,.8],[.6,.7],[.6,.9])*	*([0,.3],[.4,.7],[.2,.8])*	*([.1,.3],[.6,.8],[.3,.7])*

Table3:Tabular representation of (f_A^1,E)

The tabular representation of (f_A^2,E) is given by

U	e_1	e_2	e_3
u_1	([.4,.7],[.5,.7],[.4,.9])	([.2,.3],[.4,.5],[.7,.9])	([.3,.7],[.5,.8],[.1,.2])
u_2	([.3,.5],[.4,.8],[.1,.4])	([.4,.6],[.3,.5],[.2,.5])	([.1,.3],[.3,.5],[.6,.8])
u_3	([.3,.9],[.1,.2],[.6,.7])	([.5,.7],[.6,.7],[.3,.4])	([.2,.6],[.3,.5],[.5,.8])

Table4: Tabular representation of (f_A^2,E)

Let $(f_A^3,E)=(f_A^1,E)\cap(f_A^2,E)$ then the tabular representation of (f_A^3,E) is given by

U	e_1	e_2
u_1	([.1,.7],[.5,.8],[.4,1])	([.1,.3],[.4,.6],[.7,.9])
u_2	([.1,.3],[.6,.8],[.2,.8])	([0,.5],[.5,.8],[.4,1])
u_3	([.3,.8],[.6,.7],[.6,.9])	([0,.3],[.6,.7],[.3,.8])

e_3
([.2,.5],[.8,.9],[.4,.9])
([0,.3],[.6,.9],[.6,.8])
([.1,.3],[.6,.8],[.5,.8])

Table5:Tabular representation of $\left(f_A^3,E\right)$

Let $\left(f_A^4,E\right)=\left(f_A^1,E\right)\cup\left(f_A^2,E\right)$ then the tabular representation of $\left(f_A^4,E\right)$ is given by

U	e_1	e_2
u_1	([.4,.7],[.4,.7],[.3,.9])	([.2,.3],[.4,.5],[.2,.6])
u_2	([.3,.5],[.4,.7],[.1,.4])	([.4,.6],[.3,.5],[.2,.5])
u_3	([.4,.9],[.1,.2],[.6,.7])	([.5,.7],[.4,.7],[.2,.4])

e_3
([.3,.7],[.5,.8],[.1,.2])
([.1,.3],[.3,.5],[.1,.7])
([.2,.6],[.3,.5],[.3,.7])

Table6:Tabular representation of $\left(f_A^4,E\right)$

Here we observe that the sub-family $\tau_1=\left\{\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right),\left(f_A^1,E\right),\left(f_A^2,E\right),\left(f_A^3,E\right),\left(f_A^4,E\right)\right\}$ of $\wp\left(\zeta_A,E\right)$ is a *IVNS*-topology on $\left(\zeta_A,E\right)$, as it satisfies the necessary three axioms of topology and $\left(\zeta_A,E,\tau\right)$ is a *IVNS*-topological space. But the sub-family $\tau_2=\left\{\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right),\left(f_A^1,E\right),\left(f_A^2,E\right)\right\}$ of $\wp\left(\zeta_A,E\right)$ is not an *IVNS*-topology on $\left(\zeta_A,E\right)$, as the union $\left(f_A^4,E\right)=\left(f_A^1,E\right)\cup\left(f_A^2,E\right)$ does not belong to τ_2.

Definition 3.3: As every *IVNS*-topology on $\left(\zeta_A,E\right)$ must contains the sets $\left(\phi_{\zeta_A},E\right)$ and $\left(\zeta_A,E\right)$, so the family $\vartheta=\left\{\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right)\right\}$ forms a *IVNS*-topology on $\left(\zeta_A,E\right)$. The topology is called indiscrete *IVNS*-topology and the triplet $\left(\zeta_A,E,\vartheta\right)$ is called an indiscrete interval valued neutrosophic soft topological space (or simply indiscrete *IVNS*-topological space).

Definition 3.4: Let ξ denotes the family of all *IVNS*-subsets of $\left(\zeta_A,E\right)$. Then we observe that ξ satisfies all the axioms of topology on $\left(\zeta_A,E\right)$. This topology is called discrete interval valued neutrosophic soft topology and the triplet $\left(\zeta_A,E,\xi\right)$ is called discrete interval valued neutrosophic soft topological space (or simply discrete *IVNS*-topological space).

Theorem 3.5: Let $\left\{\tau_i : i\in I\right\}$ be any collection of *IVNS*-topology on $\left(\zeta_A,E\right)$. Then their intersection $\bigcap_{i\in I}\tau_i$ is also a *IVNS*-topology on $\left(\zeta_A,E\right)$.

Proof: (i) Since $\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right)\in\tau_i$ for each $i\in I$. Hence $\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right)\in\bigcap_{i\in I}\tau_i$.

(ii) Let $\left\{\left(f_A^k,E\right): k\in K\right\}$ be an arbitrary family of interval valued neutrosophic soft sets where $\left(f_A^k,E\right)\in\bigcap_{i\in I}\tau_i$ for each $k\in K$. Then for each $i\in I$, $\left(f_A^k,E\right)\in\tau_i$ for $k\in K$ and since for each $i\in I$, τ_i ia a *IVNS*-topology, therefore $\bigcup_{k\in K}\left(f_A^k,E\right)\in\tau_i$ for each $i\in I$. Hence $\bigcup_{k\in K}\left(f_A^k,E\right)\in\bigcap_{i\in I}\tau_i$.

(iii) Let $\left(f_A^1,E\right),\left(f_A^2,E\right)\in\bigcap_{i\in I}\tau_i$, then $\left(f_A^1,E\right),\left(f_A^2,E\right)\in\tau_i$ for each $i\in I$. Since for each $i\in I$, τ_i is an *IVNS*-topology, therefore $\left(f_A^1,E\right)\cap\left(f_A^2,E\right)\in\tau_i$ for each $i\in I$. Hence $\left(f_A^1,E\right)\cap\left(f_A^2,E\right)\in\bigcap_{i\in I}\tau_i$.

Thus $\bigcap_{i\in I}\tau_i$ satisfies all the axioms of topology. Hence $\bigcap_{i\in I}\tau_i$ forms a *IVNS*-topology. But union of *IVNS*-topologies need not be a *IVNS*-topology. Let us show this with the following example.

Example 3.6: In example 3.2, the sub families $\tau_3=\left\{\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right),\left(f_A^1,E\right)\right\}$ and $\tau_4=\left\{\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right),\left(f_A^2,E\right)\right\}$ are IVNS-topologies in $\left(\zeta_A,E\right)$. But their union $\tau_3\cup\tau_4=\left\{\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right),\left(f_A^1,E\right),\left(f_A^2,E\right)\right\}$ is not a IVNS-topology in $\left(\zeta_A,E\right)$.

Definition 3.7: Let $\left(\zeta_A,E,\tau\right)$ be an *IVNS*-topological space over $\left(\zeta_A,E\right)$. An interval valued neutrosophic soft

subset $\left(f_A, E\right)$ of $\left(\zeta_A, E\right)$ is called interval valued neutrosophic soft closed set (in short *IVNS*-closed set) if its complement $\left(f_A, E\right)^c$ is a member of τ.

Example 3.8: Let us consider example 3.2. then the IVNS-closed sets in $\left(\zeta_A, E, \tau_1\right)$ are

U	e_1	e_2	e_3
u_1	([.2,.7],[.5,.7],[.5,.8])	([.1,.3],[.7,.8],[.4,.7])	([0,.2],[.9,1],[.3,.9])
u_2	([.1,.2],[.6,.7],[.4,.7])	([.1,.2],[.8,.9],[.6,.9])	([0,.5],[.8,.9],[.4,.8])
u_3	([.3,.6],[.9,1],[.5,1])	([.1,.3],[.6,.8],[.6,.8])	([.2,.4],[.7,.9],[.4,.9])

Table7:Tabular representation of $\left(\zeta_A, E\right)^c$

U	e_1	e_2	e_3
u_1	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])
u_2	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])
u_3	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])	([1,1], [0,0],[0,0])

Table8:Tabular representation of $\left(\phi_{\zeta_A}, E\right)^c$

U	e_1	e_2	e_3
u_1	([.3,1],[.2,.6],[.1,.7])	([.2,.6],[.4,.6],[.1,.3])	([.4,.9],[.1,.2],[.2,.5])
u_2	([.2,.8],[.3,.4],[.1,.3])	([.4,1],[.2,.5],[0,.5])	([.1,.6],[.1,.4],[0,.3])
u_3	([.6,.9,[.3,.4],[.4,.8])	([.2,.8],[.3,.6],[0,.3])	([.3,.7],[.2,.4],[.1,.3])

Table9:Tabular representation of $\left(f_A^1, E\right)^c$

U	e_1	e_2	e_3
u_1	([.4,.9],[.3,.5],[.4,.7])	([.7,.9],[.5,.6],[.2,.3])	([.1,.2],[.2,.5],[.3,.7])
u_2	([.1,.4],[.2,.6],[.3,.5])	([.2,.5],[.5,.7],[.4,.6])	([.6,.8],[.5,.7],[.1,.3])
u_3	([.6,.7],[.8,.9],[.3,.9])	([.3,.4],[.3,.4],[.5,.7])	([.5,.8],[.5,.7],[.2,.6])

Table10:Tabular representation of $\left(f_A^2, E\right)^c$

U	e_1	e_2	e_3
u_1	([.4,1],[.2,.5],[.1,.7])	([.7,.9],[.4,.6],[.1,.3])	([.4,.9],[.1,.2],[.2,.5])
u_2	([.2,.8],[.2,.4],[.1,.3])	([.4,1],[.2,.5],[0,.5])	([.6,.8],[.1,.4],[0,.3])
u_3	([.6,.9],[.3,.4],[.3,.8])	([.3,.8],[.3,.4],[0,.3])	([.5,.8],[.2,.4],[.1,.3])

Table11:Tabular representation of $\left(f_A^3, E\right)^c$

U	e_1	e_2	e_3
u_1	([.3,.9],[.3,.6],[.4,.7])	([.2,.6],[.5,.6],[.2,.3])	([.1,.2],[.2,.5],[.3,.7])
u_2	([.1,.4],[.3,.6],[.3,.5])	([.2,.5],[.5,.7],[.4,.6])	([.1,.7],[.5,.7],[.1,.3])
u_3	([.6,.7],[.8,.9],[.4,.9])	([.2,.4],[.3,.6],[.5,.7])	([.3,.7],[.5,.7],[.2,.6])

Table12:Tabular representation of $\left(f_A^4, E\right)^c$

are the IVNS-closed sets in $\left(\zeta_A, E, \tau_1\right)$.

Theorem 3.9: Let $\left(\zeta_A, E, \tau\right)$ be an *IVNS*-topological space over $\left(\zeta_A, E\right)$. Then

1. $\left(\phi_{\zeta_A}, E\right)^c$, $\left(\zeta_A, E\right)^c$ are *IVNS*-closed sets.
2. Arbitrary intersection of *IVNS*-closed sets is *IVNS*-closed set.
3. Finite union of IVNS-closed sets is *IVNS*-closed set.

Proof: 1. Since $\left(\phi_{\zeta_A}, E\right), \left(\zeta_A, E\right) \in \tau$, therefore $\left(\phi_{\zeta_A}, E\right)^c, \left(\zeta_A, E\right)^c$ are *IVNS*-closed sets.

2. Let $\left\{\left(f_A^k, E\right) : k \in K\right\}$ be an arbitrary family of *IVNS*-closed sets in $\left(\zeta_A, E, \tau\right)$ and let $\left(f_A, E\right) = \bigcap_{k \in K} \left(f_A^k, E\right)$.

Now $\left(f_A,E\right)^c=\left(\bigcap_{k\in K}\left(f_A^k,E\right)\right)^c=\bigcup_{k\in K}\left(f_A^k,E\right)^c$ and $\left(f_A^k,E\right)^c\in\tau$ for each $k\in K$, so $\bigcup_{k\in K}\left(f_A^k,E\right)^c\in\tau$. Hence $\left(f_A,E\right)^c\in\tau$. Thus $\left(f_A,E\right)$ is *IVNS*-closed set.

3. Let $\left\{\left(f_A^i,E\right):i=1,2,3,...,n\right\}$ be a family of *IVNS*-closed sets in $\left(\zeta_A,E,\tau\right)$ and let $\left(g_A,E\right)=\bigcup_{i=1}^{n}\left(f_A^i,E\right)$.

Now $\left(g_A,E\right)^c=\left(\bigcup_{i=1}^{n}\left(f_A^i,E\right)\right)^c=\bigcap_{i=1}^{n}\left(f_A^i,E\right)^c$ and $\left(f_A^i,E\right)^c\in\tau$ for $i=1,2,3,...,n$, so $\bigcap_{i=1}^{n}\left(f_A^i,E\right)^c\in\tau$. Hence $\left(g_A,E\right)^c\in\tau$. Thus $\left(g_A,E\right)$ is *IVNS*-closed set.

Definition 3.10: Let $\left(\zeta_A,E,\tau_1\right)$ and $\left(\zeta_A,E,\tau_2\right)$ be two *IVNS*-topological spaces over $\left(\zeta_A,E\right)$. If each $\left(f_A,E\right)\in\tau_2$ implies $\left(f_A,E\right)\in\tau_1$, then τ_1 is called interval valued neutrosophic soft finer topology than τ_2 and τ_2 is called interval valued neutrosophic soft coarser topology than τ_1.

Example 3.11: In example 3.2 and 3.6, τ_1 is interval valued neutrosophic soft finer topology than τ_3 and τ_3 is called interval valued neutrosophic soft coarser topology than τ_1.

Definition 3.12: Let $\left(\zeta_A,E,\tau\right)$ be a *IVNS*-topological space over $\left(\zeta_A,E\right)$ and ß be a subfamily of τ. If every element of τ can be express as the arbitrary interval valued neutrosophic soft union of some elements of ß, then ß is called an interval valued neutrosophic soft basis for the *IVNS*-topology τ.

Example 3.13: In example 3.2, for the *IVNS*-topology $\tau_1=\left\{\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right),\left(f_A^1,E\right),\left(f_A^2,E\right),\left(f_A^3,E\right),\left(f_A^4,E\right)\right\}$, the subfamily ß=$\left\{\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right),\left(f_A^1,E\right),\left(f_A^2,E\right),\left(f_A^3,E\right)\right\}$ of $\wp\left(\zeta_A,E\right)$ is a interval valued neutrosophic soft basis for the *IVNS*-topology τ_1.

4 Some Properties of Interval Valued Neutrosophic Soft Topological Spaces

In this section some properties of interval valued neutrosophic soft topological spaces are introduced. Some results on IVNSInt and IVNSCl are also intoduced.

Definition 4.1: Let $\left(\zeta_A,E,\tau\right)$ be a *IVNS*-topological space and let $\left(f_A,E\right)\in IVNSS(U;E)$. The interval valued neutrosophic soft interior and closer of $\left(f_A,E\right)$ is denoted by *IVNSInt(f_A,E)* and *IVNSCl(f_A,E)* are defined as $IVNSInt\left(f_A,E\right)=\bigcup\left\{\left(g_A,E\right)\in\tau:\left(g_A,E\right)\subseteq\left(f_A,E\right)\right\}$ and $INVNSCl\left(f_A,E\right)=$ $\bigcap\left\{\left(g_A,E\right)\in\tau^c:\left(f_A,E\right)\subseteq\left(g_A,E\right)\right\}$ respectively.

Example 4.2: Let us consider example 3.2 and take an IVNSS $\left(f_A^5,E\right)$ as

U	e_1	e_2	e_3
u_1	([.2,.8],[.3,.6],[.2,.8])	([.2,.4],[.4,.6],[.2,.4])	([.2,.6],[.7,.8],[.3,.4])
u_2	([.1,.6],[.4,.5],[.2,.7])	([.2,.6],[.5,.7],[.1,.7])	([.1,.4],[.2,.5],[.1,.5])
u_3	([.5,.8],[.5,.6],[.5,.8])	([.1,.4],[.4,.6],[.1,.5])	([.2,.5],[.5,.8],[.2,.4])

Table13:Tabular representation of $\left(f_A^5,E\right)$

Now $IVNSInt\left(f_A^5,E\right)=\left(f_A^1,E\right)$ and $IVNSCl\left(f_A^5,E\right)=\left(f_A^1,E\right)^c$.

Theorem 4.3: Let $\left(\zeta_A,E,\tau\right)$ be a *IVNS*-topological space and $\left(f_A,E\right)$, $\left(g_A,E\right)\in IVNSS\left(U;E\right)$ then the following properties hold

1. $IVNSInt\left(f_A,E\right)\subseteq\left(f_A,E\right)$
2. $\left(f_A,E\right)\subseteq\left(g_A,E\right)\Rightarrow IVNSInt\left(f_A,E\right)\subseteq IVNSInt\left(g_A,E\right)$
3. $IVNSInt\left(f_A,E\right)\in\tau$
4. $\left(f_A,E\right)\in\tau\Leftrightarrow IVNSInt\left(f_A,E\right)=\left(f_A,E\right)$
5. $IVNSInt\left(IVNSInt\left(f_A,E\right)\right)=IVNSInt\left(f_A,E\right)$
6. $IVNSInt\left(\phi_A,E\right)=\phi_A$, $IVNSInt\left(U_A,E\right)=U_A$

Proof:

1. Straight forward.

2. $\left(f_A,E\right)\subseteq\left(g_A,E\right)$ implies all the *IVNS*-open sets contained in $\left(f_A,E\right)$ also contained in $\left(g_A,E\right)$.

i.e.

$$\left\{\left(f_A^*,E\right)\in\tau:\left(f_A^*,E\right)\subseteq\left(f_A,E\right)\right\}\subseteq\left\{\left(g_A^*,E\right)\in\tau:\left(g_A^*,E\right)\subseteq\left(g_A,E\right)\right\}$$

i.e.

$\cup\left\{\left(f^*_A,E\right)\in\tau:\left(f^*_A,E\right)\subseteq\left(f_A,E\right)\right\}\subseteq\cup\left\{\left(g^*_A,E\right)\in\tau:\left(g^*_A,E\right)\subseteq\left(g_A,E\right)\right\}$

i.e. $IVNSInt\left(f_A,E\right)\subseteq IVNSInt\left(g_A,E\right)$

3. $IVNSInt\left(f_A,E\right)=\cup\left\{\left(f^*_A,E\right)\in\tau:\left(f^*_A,E\right)\subseteq\left(f_A,E\right)\right\}$

It is clear that $\cup\left\{\left(f^*_A,E\right)\in\tau:\left(f^*_A,E\right)\subseteq\left(f_A,E\right)\right\}\in\tau$

So, $IVNSInt\left(f_A,E\right)\in\tau$.

4. Let $\left(f_A,E\right)\in\tau$, then by (1) $IVNSInt\left(f_A,E\right)\subseteq\left(f_A,E\right)$.

Now since $\left(f_A,E\right)\in\tau$ and $\left(f_A,E\right)\subseteq\left(f_A,E\right)$, Therefore

$\left(f_A,E\right)\subseteq\cup\left\{\left(g^*_A,E\right)\in\tau:\left(g^*_A,E\right)\subseteq\left(g_A,E\right)\right\}=IVNSInt\left(f_A,E\right)$

i.e, $\left(f_A,E\right)\subseteq IVNSInt\left(f_A,E\right)$

Thus $IVNSInt\left(f_A,E\right)=\left(f_A,E\right)$

Conversly, let $IVNSInt\left(f_A,E\right)=\left(f_A,E\right)$

Since by (3) $IVNSInt\left(f_A,E\right)\in\tau$

Therefore $\left(f_A,E\right)\in\tau$

5. By (3) $IVNSInt\left(f_A,E\right)\in\tau$

$\therefore$ By (4) $IVNSInt\left(IVNSInt\left(f_A,E\right)\right)=IVNSInt\left(f_A,E\right)$.

6. We know that $\left(\phi_A,E\right),\left(U_A,E\right)\in\tau$

$\therefore$ By (4) $IVNSInt\left(\phi_A,E\right)=\phi_A$, $IVNSInt\left(U_A,E\right)=U_A$

Theorem 4.4: Let $\left(\zeta_A,E,\tau\right)$ be a *IVNS*-topological space and $\left(f_A,E\right)$, $\left(g_A,E\right)\in IVNSs\left(U;E\right)$ then the following properties hold

1. $\left(f_A,E\right)\subseteq IVNSCl\left(f_A,E\right)$
2. $\left(f_A,E\right)\subseteq\left(g_A,E\right)\Rightarrow IVNSCl\left(f_A,E\right)\subseteq IVNSCl\left(g_A,E\right)$
3. $\left(IVNSCl\left(f_A,E\right)\right)^c\in\tau$
4. $\left(f_A,E\right)^c\in\tau\Leftrightarrow IVNSCl\left(f_A,E\right)=\left(f_A,E\right)$
5. $IVNSCl\left(IVNSCl\left(f_A,E\right)\right)=IVNSCl\left(f_A,E\right)$
6. $IVNSCl\left(\phi_A,E\right)=\phi_A$, $IVNSCl\left(U_A,E\right)=U_A$

Proof: straight forward.

Theorem 4.5: Let $\left(\zeta_A,E,\tau\right)$ be an *IVNS*-topological space on $\left(\zeta_A,E\right)$ and let $\left(f_A,E\right),\left(g_A,E\right)\in IVNSs\left(U;E\right)$. Then the following properties hold

1. $IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)=IVNSInt\left(f_A,E\right)\cap IVNSInt\left(g_A,E\right)$
2. $IVNSInt\left(\left(f_A,E\right)\cup\left(g_A,E\right)\right)\supseteq IVNSInt\left(f_A,E\right)\cup IVNSInt\left(g_A,E\right)$
3. $IVNSCl\left(\left(f_A,E\right)\cup\left(g_A,E\right)\right)=IVNSCl\left(f_A,E\right)\cup IVNSCl\left(g_A,E\right)$
4. $IVNSCl\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)\subseteq IVNSCl\left(f_A,E\right)\cap IVNSCl\left(g_A,E\right)$
5. $\left(IVNSInt\left(f_A,E\right)\right)^c=IVNSCl\left(f_A,E\right)^c$
6. $\left(IVNSCl\left(f_A,E\right)\right)^c=IVNSInt\left(f_A,E\right)^c$

Proof:

1. By theorem 4.2 (1), $IVNSInt\left(f_A,E\right)\subseteq\left(f_A,E\right)$ and $IVNSInt\left(g_A,E\right)\subseteq\left(g_A,E\right)$. Thus $IVNSInt\left(f_A,E\right)\cap IVNSInt\left(g_A,E\right)\subseteq\left(f_A,E\right)\cap\left(g_A,E\right)$.

Hence

$IVNSInt\left(f_A,E\right)\cap IVNSInt\left(g_A,E\right)\subseteq IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)$ …………… (i)

Again since $\left(f_A,E\right)\cap\left(g_A,E\right)\subseteq\left(f_A,E\right)$. By theorem 4.2 (2), $IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)\subseteq IVNSInt\left(f_A,E\right)$.

Similarly

$IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)\subseteq IVNSInt\left(g_A,E\right)$

Hence

$IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)\subseteq IVNSInt\left(f_A,E\right)\cap IVNSInt\left(g_A,E\right)$ … …………… (ii)

Using (i) and (ii) we get, $IVNSInt\left(\left(f_A,E\right)\cap\left(g_A,E\right)\right)=IVNSInt\left(f_A,E\right)\cap IVNSInt\left(g_A,E\right)$.

2. Since $\left(f_A,E\right)\subseteq\left(f_A,E\right)\cup\left(g_A,E\right)$.

By theorem 4.2 (2), $IVNSInt\left(f_A,E\right)\subseteq IVNSInt\left(\left(f_A,E\right)\cup\left(g_A,E\right)\right)$.

Similarly, $IVNSInt\left(g_A,E\right)\subseteq IVNSInt\left(\left(f_A,E\right)\cup\left(g_A,E\right)\right)$.

Hence $IVNSInt\left(\left(f_A,E\right)\cup\left(g_A,E\right)\right)\supseteq IVNSInt\left(f_A,E\right)\cup IVNSInt\left(g_A,E\right)$.

3. Similar to 1.

4. Similar to 2.

5. $\left(IVNSInt\left(f_A,E\right)\right)^c=\left(\cup\left\{\left(g_A,E\right)\in\tau:\left(g_A,E\right)\subseteq\left(f_A,E\right)\right\}\right)^c$

$$=\cap\left\{\left(g_A,E\right)\in\tau^c:\left(f_A,E\right)^c\subseteq\left(g_A,E\right)\right\}$$

$$=IVNSCl\left(f_A,E\right)^c$$

6. Similar to 5.

Equality does not hold in theorem 4.4 (2), (4). Let us show this by an example.

Example 4.6: Let $U=\{u_1,u_2\}$, $E=\{e_1,e_2,e_3\}$, $A=\{e_1,e_2\}$. The tabular representation of (ζ_A,E) is given by

U	e1	e2
u_1	([.5,.8],[.3,.5],[.2,.7])	([.3,.9],[.1,.2],[0,.1])
u_2	([.4,.6],[.3,.4],[.1,.2])	([.4,.8],[.1,.3],[.1,.2])

Table14:Tabular representation of (ζ_A,E)

The tabular representation of (ϕ_{ζ_A},E) is given by

U	e1	e2
u_1	([0,0], [1,1], [1,1])	([0,0], [1,1], [1,1])
u_2	([0,0], [1,1], [1,1])	([0,0], [1,1], [1,1])

Table15:Tabular representation of (ϕ_{ζ_A},E)

The tabular representation of (f_A,E) is given by

U	e1	e2
u_1	([.1,.7],[.4,.8],[.3,1])	([.2,.5],[.7,.9],[.3,.7])
u_2	([.1,.2],[.6,.7],[.2,.7])	([0,.3],[.5,.8],[.4,1])

Table16:Tabular representation of (f_A,E)

Clearly $\tau=\{(\phi_{\zeta_A},E),(\zeta_A,E),(f_A,E)\}$ is a IVNS-topology on (ζ_A,E) . Let us now take two interval valued neutrosophic soft sets (g_A,E) and (h_A,E) as

U	e1	e2
u_1	([.1,.6],[.4,.9],[.4,1])	([.1,.5],[.7,.9],[.3,.8])
u_2	([.1,.2],[.6,.7],[.2,.8])	([0,.2],[.5,.9],[.4,1])

Table17:Tabular representation of (g_A,E)

U	e1	e2
u_1	([0,.7],[.5,.8],[.3,1])	([.2,.5],[.8,1],[.6,.7])
u_2	([.1,.2],[.6,.8],[.3,.7])	([0,.3],[.6,.8],[.5,1])

Table18:Tabular representation of (h_A,E)

Now $(g_A,E)\cup(h_A,E)=(f_A,E)$

$\therefore$

$IVNSInt((g_A,E)\cup(h_A,E))=IVNSInt(f_A,E)=(f_A,E)$

Also $IVNSInt(g_A,E)=(\phi_{\zeta_A},E)$, $IVNSInt(h_A,E)=(\phi_{\zeta_A},E)$

$\therefore$

$IVNSInt(g_A,E)\cup IVNSInt(h_A,E)=(\phi_{\zeta_A},E)\cup(\phi_{\zeta_A},E)=(\phi_{\zeta_A},E)$

Thus

$IVNSInt((f_A,E)\cup(g_A,E))\neq IVNSInt(f_A,E)\cup IVNSInt(g_A,E)$.

Therefore equality does not hold for (2).

By theorem 4.4 (5),

$IVNSCl(g_A,E)^c=(IVNScl(g_A,E))^c=(\phi_{\zeta_A},E)^c=(\zeta_A,E)$.

Similarly $IVNSCl(h_A,E)^c=(\zeta_A,E)$.

Therefore

$IVNSCl(g_A,E)^c\cap IVNSCl(h_A,E)^c=(\zeta_A,E)\cap(\zeta_A,E)=(\zeta_A,E)$. Also

$$IVNSCl((g_A,E)^c\cap(h_A,E)^c)=IVNSCl((g_A,E)\cup(h_A,E))^c$$
$$=(IVNSInt((g_A,E)\cup(h_A,E)))^c$$
$$=(IVNSInt(f_A,E))^c$$
$$=(f_A,E)^c$$

Thus

$IVNSCl((f_A,E)\cap(g_A,E))\neq IVNSCl(f_A,E)\cap IVNSCl(g_A,E)$. Therefore equality doesnot hold in (4).

5 Interval Valued Neutrosophic Soft Subspace Topology

In this section we introduce the concept of interval valued neutrosophic soft subspace topology along with some examples and results.

Theorem 5.1: Let (ζ_A,E,τ) be an *IVNS*-topological space on (ζ_A,E) and $(f_A,E)\in\wp(\zeta_A,E)$. Then the collection $\tau_{(f_A,E)}=\{(f_A,E)\cap(g_A,E):(g_A,E)\in\tau\}$ is an *IVNS*-topology on (ζ_A,E) .

Proof:

(i) Since $(\phi_{\zeta_A},E),(\zeta_A,E)\in\tau$, therefore $(f_A,E)\cap(\phi_{\zeta_A},E)=(\phi_{f_A},E)\in\tau_{(f_A,E)}$ and $(f_A,E)\cap(\zeta_A,E)=(f_A,E)\in\tau_{(f_A,E)}$.

(ii) Let $(f_A^k,E)\in\tau_{(f_A,E)},\forall k\in K$ .Then $(f_A^k,E)=(f_A,E)\cap(g_A^k,E)$ where $(g_A^k,E)\in\tau$ for each $k\in K$.

Now

$\bigcup_{k\in K}(f_A^k,E)=\bigcup_{k\in K}((f_A,E)\cap(g_A^k,E))=(f_A,E)\cap\left(\bigcup_{k\in K}(g_A^k,E)\right)\in\tau_{(f_A,E)}$

(since $\bigcup_{k\in K}(g_A^k,E)\in\tau$ as each $(g_A^k,E)\in\tau$.

(iii) Let $(f_A^1,E),(f_A^2,E)\in\tau_{(f_A,E)}$ then $(f_A^1,E)=(f_A,E)\cap(g_A^1,E)$ and $(f_A^2,E)=(f_A,E)\cap(g_A^2,E)$ where $(g_A^1,E),(g_A^2,E)\in\tau$.

Now

$$\left(f_A^1,E\right)\cap\left(f_A^2,E\right)=\left(\left(f_A,E\right)\cap\left(g_A^1,E\right)\right)\cap\left(\left(f_A,E\right)\cap\left(g_A^2,E\right)\right)$$
$$=\left(f_A,E\right)\cap\left(\left(g_A^1,E\right)\cap\left(g_A^2,E\right)\right)\in\tau_{\left(f_A,E\right)}$$

(since $\left(g_A^1,E\right)\cap\left(g_A^2,E\right)\in\tau$ as $\left(g_A^1,E\right),\left(g_A^2,E\right)\in\tau$).

Definition 5.2: Let $\left(\zeta_A,E,\tau\right)$ be an *IVNS*-topological space on $\left(\zeta_A,E\right)$ and $\left(f_A,E\right)\in\wp\left(\zeta_A,E\right)$. Then the IVNS-topology $\tau_{\left(f_A,E\right)}=\left\{\left(f_A,E\right)\cap\left(g_A,E\right):\left(g_A,E\right)\in\tau\right\}$ is called interval valued neutrosophic soft subspace topology and $\left(f_A,E,\tau_{\left(f_A,E\right)}\right)$ is called interval valued neutrosophic soft subspace of $\left(\zeta_A,E,\tau\right)$.

Example 5.3: Let us consider the IVNS-topology $\tau_1=\left\{\left(\phi_{\zeta_A},E\right),\left(\zeta_A,E\right),\left(f_A^1,E\right),\left(f_A^2,E\right),\left(f_A^3,E\right),\left(f_A^4,E\right)\right\}$ as in example 3.2 and an *IVNSS* $\left(f_A,E\right)$:

U	e_1	e_2	e_3
u_1	([.4,.6],[.6,.7],[.3,.5])	([.5,.7],[.4,.6],[0,.3])	([.3,.5],[.5,.8],[.2,.3])
u_2	([.2,.3],[.3,.6],[.5,.7])	([.6,.8],[.4,.5],[.2,.3])	([.5,.8],[.5,.7],[.2,.3])
u_3	([.5,.7],[.4,.6],[.3,.4])	([.4,.5],[.7,.9],[.6,.7])	([.1,.3],[.7,.9],[.5,.7])

Table19:Tabular representation of $\left(f_A^1,E\right)$

Then $\left(\phi_{f_A},E\right)=\left(f_A,E\right)\cap\left(\phi_{\zeta_A},E\right)$:

U	e_1	e_2	e_3
u_1	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])
u_2	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])
u_3	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])	([0,0],[1,1],[1,1])

Table20:Tabular representation of $\left(\phi_{f_A},E\right)$

$\left(g_A^1,E\right)=\left(f_A,E\right)\cap\left(f_A^1,E\right)$:

U	e_1	e_2	e_3
u_1	([.1,.6],[.6,.7],[.3,1])	([.1,.3],[.4,.6],[.2,.6])	([.2,.5],[.5,.8],[.4,.9])
u_2	([.1,.3],[.6,.7],[.5,.8])	([0,.5],[.4,.5],[.4,1])	([0,.3],[.6,.9],[.2,.7])
u_3	([.4,.7],[.4,.6],[.6,.9])	([0,.3],[.7,.9],[.6,.8])	([.1,.3],[.7,.9],[.5,.7])

Table21:Tabular representation of $\left(g_A^1,E\right)$

$\left(g_A^2,E\right)=\left(f_A,E\right)\cap\left(f_A^2,E\right)$:

U	e_1	e_2	e_3
u_1	([.4,.6],[.6,.7],[.4,.9])	([.2,.3],[.4,.6],[.7,.9])	([.3,.5],[.5,.8],[.2,.3])
u_2	([.2,.3],[.4,.8],[.5,.7])	([.4,.6],[.4,.5],[.2,.5])	([.1,.3],[.5,.7],[.6,.8])
u_3	([.3,.7],[.4,.6],[.6,.7])	([.4,.5],[.7,.9],[.6,.7])	([.1,.3],[.7,.9],[.3,.8])

Table22:Tabular representation of $\left(g_A^2,E\right)$

$\left(g_A^3,E\right)=\left(f_A,E\right)\cap\left(f_A^3,E\right)$:

U	e_1	e_2	e_3
u_1	([.1,.6],[.6,.8],[.4,1])	([.1,.3],[.4,.6],[.7,.9])	([.2,.5],[.5,.8],[.4,.9])
u_2	([.1,.3],[.6,.8],[.5,.8])	([0,.5],[.4,.5],[.4,1])	([0,.3],[.6,.9],[.6,.8])
u_3	([.3,.7],[.4,.6],[.6,.9])	([0,.3],[.7,.9],[.6,.8])	([.1,.3],[.7,.9],[.5,.8])

Table23:Tabular representation of $\left(g_A^3,E\right)$

$\left(g_A^4,E\right)=\left(f_A,E\right)\cap\left(f_A^4,E\right)$:

U	e_1	e_2	e_3
u_1	([.2,.5],[.5,.8],[.4,.9])	([.2,.5],[.5,.8],[.4,.9])	([.3,.5],[.5,.8],[.2,.3])
u_2	([0,.3],[.6,.9],[.6,.8])	([0,.3],[.6,.9],[.6,.8])	([.1,.3],[.5,.7],[.2,.7])
u_3	([.1,.3],[.7,.9],[.5,.8])	([.1,.3],[.7,.9],[.5,.8])	([.1,.3],[.7,.9],[.5,.7])

Table24:Tabular representation of $\left(g_A^4,E\right)$

Then $\tau_{\left(f_A,E\right)}=\left\{\left(\phi_{f_A},E\right),\left(f_A,E\right),\left(g_A^1,E\right),\left(g_A^2,E\right),\left(g_A^4,E\right)\right\}$ is an interval valued neutrosophic soft subspace

topology for τ_1 and $\left(f_A, E, \tau_{(f_A,E)}\right)$ is called interval valued neutrosophic soft subspace of $\left(\zeta_A, E, \tau_1\right)$.

Theorem 5.4: Let $\left(\zeta_A, E, \tau\right)$ be an *IVNS*-topological space on $\left(\zeta_A, E\right)$, ß be an *IVNS*-basis for τ and $\left(f_A, E\right) \in \wp\left(\zeta_A, E\right)$. Then the family $ß_{(f_A,E)} = \left\{\left(f_A, E\right) \cap \left(g_A, E\right) : \left(g_A, E\right) \in ß\right\}$ is an *IVNS*-basis for subspace topology $\tau_{(f_A,E)}$.

Proof: Let $\left(h_A, E\right) \in \tau_{(f_A,E)}$ be arbitrary, then there exists an *IVNSS* $\left(g_A, E\right) \in \tau$ such that $\left(h_A, E\right) = \left(f_A, E\right) \cap \left(g_A, E\right)$. Since ß is a basis for τ, therefore there exists a sub collection $\left\{\left(\chi_A^i, E\right) : i \in I\right\}$ of ß such that $\left(g_A, E\right) = \bigcup_{i\in I}\left(\chi_A^i, E\right)$.

Now

$$\left(h_A, E\right) = \left(f_A, E\right) \cap \left(g_A, E\right) = \bigcup_{i\in I}\left(\chi_A^i, E\right) = \bigcup_{i\in I}\left(\left(f_A, E\right) \cap \left(\chi_A^i, E\right)\right)$$

. Since $\left(f_A, E\right) \cap \left(\chi_A^i, E\right) \in ß_{(f_A,E)}$, therefore $ß_{(f_A,E)}$ is an *IVNS*-basis for the subspace topology $\tau_{(f_A,E)}$.

Conclusion

In this paper we introduce the concept of interval valued neutrosophic soft topology. Some basic theorem and properties of the above concept are also studied. IVN interior and IVN closer of an interval valued neutrosophic soft set are also defined. Interval valued neutrosophic soft subspace topology is also studied.

In future there will be more research work in this concept, taking the basic definitions and results from this article**.**

References

[1] H. . Aktas , N. Cagman, Soft Sets and soft groups, Inform. Sci., 177(2007), 2726-2735.

[2] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications 57(9)(2009),1547-1553.

[3]I. Arockiarani, I. R. Sumathi, J. Martina Jency, “Fuzzy neutrosophic soft topological spaces, International Journal of Mathematical Archive,4(10)(2013), 225-238.

[4] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87-96.

[5] S. Broumi, I. Deli, and F. Smarandache, Relations on Interval Valued Neutrosophic Soft Sets, Journal of New Results in Science, 5 (2014), 1-20.

[6] I. Deli, Interval-valued neutrosophic soft sets and its decision making, Kilis 7 Aralık University, 79000 Kilis, Turkey.

[7] P. K. Maji, R. Biswas and A. R. Roy, Soft Set Theory, Computers and Mathematics with Applications, 45(2003),555-562.

[8] P. K. Maji, Neutrosophic soft set, Annals of Fuzzy Mathematics and Information, 5(1)(2013), 157-168.

[9] D. Molodtsov, Soft set theory-first results, Computers and Mathematics with Applications 37(4-5)(1999), 19-31.

[10] Anjan Mukherjee, Ajoy Kanti Das, Abhijit Saha, Interval valued intuitionistic fuzzy soft topological spaces, Annals of Fuzzy Mathematics and Informatics, 6 (3), (2013), 689-703.

[11] D. Pie, D. Miao, From soft sets to information systems, Proc. IEEE Int. Conf. Granular Comput. 2(2005), 617-621.

[12] F. Smarandache, Neutrosophic Logic and Set, mss., http://fs.gallup.unm.edu/neutrosophy.htm, 1995.

[13] F. Smarandache, Neutrosophic set- a generalisation of the intuitionistic fuzzy sets, int. J. Pure Appl. Math. 24(2005), 287–297.

[14] H. Wang, F. Smarandache, Y.Q. Zhang, R. Sunderraman, Interval Neutrosophic Sets and logic: Theory and Applications in Computing, Hexis; Neutrosophic book series, No: 5, 2005

[15] L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965), 338-353.

Received: September 30, 2014. Accepted: October 25, 2014.